Challenging pattern tasks with geometric transformations in elementary teachers training: tessellations, polygons patterns and kaleidoscopes

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Abstract

Studying mathematics through patterns is an opportunity for students of all levels to develop mathematical knowledge connecting different subjects like geometry/geometric transformations and algebra. In a teacher training course an exploratory study was developed in the context of two disciplines: one of mathematics and other of mathematics didactics. Pre-service teachers worked with challenging tasks looking for geometric transformations that underlie the patterns constructed. According to pre-service teachers the options made in this teacher training course seem to be adequate to the challenge future teachers will face in schools.

Resumé

Étudier les mathématiques à travers des structures est une occasion pour les étudiants de tous les niveaux de développer la connaissance mathématique en reliant différents sujets comme la géométrie/les transformations géométriques et l'algèbre. Dans un cours de formation de professeurs on a mené une étude exploratoire dans le contexte de deux disciplines: les mathématiques et la didactique. Les étudiants du cours de professeurs on travaillé avec des tâches défiante en recherchant les transformations géométriques qui sont à la base des structures construites. Selon les étudiants, les options choisies dans ce cours de formation de professeurs semblent être adéquates au défi que les futurs professeurs trouveront dans les écoles.

Keywords: patterns, geometric knowledge, geometric transformations, teacher training.

Considered mathematics as the science of patterns (Biggs e Shaw, 1985; Devlin, 2002; Goldenberg, 1998; Mottershead, 1985; Orton, 1999) it was the start point for our work. Patterns gave many opportunities for the study of mathematical concepts and the development of mathematical process as problem solving, communication, reasoning and proof.

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At Portugal we have new national mathematics programme at elementary levels, (grades 1-9) where patterns, algebraic thinking and geometric transformations have a more emphasis and a change of a paradigm. So it is important that mathematics education pay attention to this changes. In this study we are concerned with patterns and geometric transformations.

The most widespread connection of patterns with mathematical concepts it is at algebraic field. At elementary school algebra is approached through recognition of pattern and consequent looking for generalization in an informal or formal way depending on the level of students. Most of the patterns studied are those that we call growing pattern.

The type of pattern that we are now interesting are someway different from those that we studied to develop algebraic thinking: the geometric patterns. Patterns that we can find everywhere around us from flowers to building, passing through clothes and make use of concepts and properties of geometric transformations.

To provide a definition of pattern it is difficult. We will used that from Sawyer (1995) when he says that a pattern is any kind of regularity that can be recognized by the mind. But when we study geometric pattern the idea of repetition it is important, but is not enough, as Jean Orton (1999) refer, it includes ideas about shapes recognition, congruence and similarity. Includes also another idea from Zusne (1975) that this kind of pattern is like a configuration consisting of several elements that somehow belong together.

Teacher has an important role in the classroom as mediator between students and knowledge. They need to organize the teaching and learning in a way that they can get involved in mathematics. As patterns are transversal features of mathematics, children need to work with them in all contents. Pre-service teachers need to study and work with patterns, to develop competences in this subject in their teacher training course to do the same in their classrooms and to aid students to observe, to conjecture, to investigate, to communicate, to recognize invariants and look for patterns (Hefendehl-Hebeker, 1998).

The exploratory study

If it is our understanding of mathematics as the science of patterns and as they appear in all mathematical contents pre-service teachers need to work with patterns in different subjects with different perspectives, since mathematical to didactical ones.

Geometry is an important subject to be worked in schools because it allows us to perceive our real world, it’s necessary also to interplay between concrete and abstract side of geometry and this is a challenge to a teacher. Besides our students have low performance levels in items related to geometry. Visualization in mathematics is in renaissance, but little pedagogical efforts seems to be invested to implement it. Like Hershkowitz (1998) said, may be that some mathematics teachers possess a naive assumption that the human beings are born with visual thinking abilities that are used when necessary and school don’t need to develop them.
Dreyfus (1990) says that visualization plays an important role to student reasoning. Besides this visual reasoning has low status because it is assumed as a preliminary stage of reasoning process. So, mathematics learning must include programmes that compel students to think visually and they can develop this ability through experiences in situations that require such kind of thinking. To foster visualization it is needed to develop the “geometrical eye” of students and teachers (Fujita & Jones, 2002).

Nowadays in Portugal, as it was referred before, we have a new Mathematics National Programme (ME, 2007) that emphasizes the study of rigid motions/geometric transformations since early grades (grade 1-2). To implement this programme pre-service and in-service teachers need to be prepared in this subjects. As teachers educators, in an elementary teacher training course, we must be aware to these new approaches of our programme, so future teachers prepare their own students with solid mathematical knowledge for their critic adult roles in an information-rich society.

We design an exploratory study which intends to describe and to analyze the work developed by pre-service teachers when they construct and look for patterns in rigid motion/geometric transformations. In particular we want: (a) to promote the development of mathematics knowledge on geometric transformations and its didactics for elementary grades (1-6); (b) to develop competences in observing; noticing patterns; conjecturing; testing and refining conjectures; justifying; proving; and (c) to adapt and construct didactical materials to explore this subjects. According to these goals we focus on a exploratory qualitative approach. Data has been collected through problem solving tasks, observations and interviews. Data analysis was in a holistic, descriptive and interpretative way.

Participants and context. The study we will present is included in a large research project Mathematics and Patterns: perspectives and curricular experiences of students and teachers. The project is being developed with two main groups: students (including pre-service teachers) and teachers. We work with some students of pre-service teacher training course (grades 1-6) at their last year of mathematics preparation in two disciplines, Geometric Transformations and Didactics of Mathematics. In these disciplines we want to promote the development of mathematical and didactical knowledge, particularly on geometric transformation, and of competences in observing and noticing patterns, conjecturing, testing and refining conjectures, justifying and proving. To achieve these goals we need to organize a learning environment so that pre-service teachers can solve and explore problem tasks, and also to get involved in class discussions. During some classes was used an application of dynamic geometry. This one made possible that features which maintain invariant, gain more easily relevance and may contribute to

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discover patterns and to formulate conjectures. After that students need to understand why the conjectures are true and then, proof could appear plain of significance.

*Some examples of tasks proposed.* In the mathematics discipline of *Geometric Transformations* pre-service teachers began to study the *geometric transformations* like translations, rotations, reflections and glide reflections. They worked with the definitions and explored and analyzed properties of the transformations. After this kind of work they were challenged with the following task: “Imagine you are a designer in a factory that makes wallpaper. A client wants a new wallpaper made by a motif he chooses. Please create new wallpaper patterns proposals with this motif.” This is the motif brought by the client. The pre-service teachers created different wallpaper patterns and were asked to identify the geometric transformations that underlie each one of the patterns created.

To do so they need to look to the patterns and “see” the geometric transformations, that are not always explicit. This kind of task needs, in the sense of Fujita & Jones (2002), the “geometrical eye” and develops visualization competences of the pre-service teachers. Firstly pre-service teachers looks to the patterns using a global apprehension of the figures, but after that they went on the analysis using a punctual apprehension of the figures, relating the original point and the corresponding transformation point. Pre-service teachers identify different kind of geometric transformations, like translations, rotations, reflections and glide reflections, as is shown in figures.

Secondly, they identified too some of their compositions, for example, the composition of two half turns with different centres, the composition of two reflections of parallels axis, the composition of two reflections of concurrent axis and they conjecture about the existence, in each
case, of just one geometric transformation equivalent of the compositions. To test the conjectures with more examples, pre-service teachers used a environment of dynamic geometry (Geometer’s Sketchpad). After testing the conjectures they looked for a proof to each case.

When pre-service teachers look for features that repeat in a problem; when they intend to understand the reasons for the repetition and observe the relations that maintain invariant when all around changes; when they intend to understand and explain the reasons that justify these relations; then they develop their mathematics comprehension and competences in a way close that of mathematicians. Pre-service teachers also develop a more dynamic conception about mathematics and they find out that patterns are underlying to all mathematics.

In the discipline of Didactics of Mathematics, after the initial study of geometric transformations in the mathematics discipline, pre-service teachers have to look for this topic in a didactic perspective for 1-6 grades students. So they were firstly challenged to explore real images of embroideries, pottery, floor and wall tiles, tessellations, friezes and to identify and to explore the geometric transformations involved. The goal of these kind of tasks was to aware pre-service teachers that, like Orton (1999) said, geometric patterns are present in everyday life. They also analyze a didactical learning sequence for study geometric transformations and tasks that can be used with elementary schools students. We can illustrate some examples used during these classes.

Pre-service teachers are asked to construct a motif, that must not have reflection axes, using a square tile. After they must to make friezes and tessellations and identify the geometric transformations that underlie the pattern constructed.

The figures show one student proposal presented. The motif chosen was made in a transparency to be more easy to verify the geometric transformations used. This kind of material revealed more easier to work with young children because of it versatility.

Another proposal it is to investigate the rotational symmetries of the geometric forms bellow. The exploration presents a very good way to work we elementary school students. With this materials they can analyze more easily the existence of rotations with, for example, a quarter turn and half turn because the geometric forms were inscribed in a “portractor”.
Pre-service teachers are also challenged to work with pentominoes. First of all they solve the follow problem: “To the party of school it will be need to put together five small squares tables. Show the different arrays of putting together the tables, knowing that two squares need to have a common side”. To solve this problem pre-service teachers constructed all pentominoes. Observed the twelve pieces and discovered which kind of symmetry has each pentominoes.

Tessellations offer rich classroom interactions that allow students to realize, for instance, that the tiled floor, their desk rest on, is a tiled plane. The mathematics associated with geometric transformations further connects students to their mathematics education. So this content provide a rich opportunity for pre-service teachers to connect and motivate geometric learning in their future classrooms. So pre-service teachers were asked if all the pentominoes tessellate the plane. The answer is “No”. They thought that only pentominoes like “I or X” do it. To home work they were asked to experiment if they can get a tessellation using each one of different pentaminoes. They were enthusiastic because they concluded that all pentominoes could tessellate the plane and they conclude also that it’s possible to put together the same pieces in different ways, as it is shown in figures.

After that they look for geometric transformations used in each of their tessellation.
The pre-service teachers also constructed *polygons patterns* gluing different colored paper rectangle. The polygons patterns are generated using the principles: (a) a simple rule is used to overlap the rectangles; (b) the angle of overlap rotation of the consecutive rectangles is a factor of 360° (so that after a fixed number of rectangles the pattern is complete and symmetrical).

The figure represents four polygon patterns constructed with rectangles. Each new rectangle is overlapped so that one of its diagonals coincides with one diagonal of the previous rectangle.

After that, by observing these polygons patterns it is possible to explore geometrical relations between several shapes that emerge, like classifying polygons, analyzing the congruence or similarity of triangles, quadrilaterals, hexagons, and so on. Moreover of these concepts related to geometric transformations it’s possible to explore, for example, counting, area and perimeter.

To apply the knowledge acquired we proposed to pre-service the construction of a *kaleidoscope*. Nowadays are available several types of kaleidoscopes that delight us when we turn it and look. They are made with mirrors, triangular prismatic mirrors and a lens multifaceted that originate several and complex images. We proposed to our pre-service teachers the constructions of simple kaleidoscope. They used reflector paper to construct triangular prisms. Those triangles could be equilateral or isosceles and acute or obtuse ones. The figures show different phases of the construction of an kaleidoscope.
We propose this task because kaleidoscopes are applications of the reflections and constitute an attractive material for elementary school students construct and explore these subjects.

We thought that the tasks presented in this teacher training course allowed pre-service to be more confident to face similar tasks with their own future students because they experimented the doubts, the uncertainty, the feeling of not to know the answer and not to know how to explore the situations presented and they gained more confidence in their own knowledge to face “new” and challenging situations.

Some conclusion

After this exploratory study and with preliminary results it’s possible to say that pre-service teachers revealed more interest to explore geometric transformation in the context of frieze and wall paper patterns they construct by their own, because this is a “challenging task” and become more confident and aware to face the implementation of the new mathematics programme to elementary school; they learned more deeply to recognize geometric transformations and their compositions in a context; despite they revealed some difficulties in visualizing “I have difficulties in visualizing; at the beginning I need to use the motif in paper and turn, flip and slide it to “see” if the geometric transformations I had imagine are or not real” and in justifying and constructing proofs for their own conjectures, they defend that “it is important get involved in tasks like these ... to make proofs because despite proving is a very difficult task ... how can I explain this ... proof compel us to think a lot and we will be more conscious about our knowledge and our difficulties”.

We expect that our instruction with pre-service teachers it will be a way for they follow and engage youngsters in their classrooms. According to pre-service teachers opinions the options made in this teacher training course are adequate to the challenge future teachers will face in schools. They defended that they can use the learning sequence proposed for this subject with his own children’s in elementary schools.

References


